

## A SUSY $A_4$ model for fermion masses and mixings

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**ABSTRACT:** We study a supersymmetric extension of the Standard Model based on discrete  $A_4 \times Z_3 \times Z_4$  flavor symmetry. We obtain quark mixing angles as well as a realistic fermion mass spectrum and we predict tribimaximal leptonic mixing by a spontaneous breaking of  $A_4$ . The top quark Yukawa interaction is present at the renormalizable level in the superpotential while all the other Yukawa interactions arise only at higher orders. We study the Higgs potential and show that it can potentially solve the so called vacuum alignment problem. The leading order predictions are not spoiled by subleading corrections.

**KEYWORDS:** Discrete and Finite Symmetries, Neutrino Physics, Beyond Standard Model, Quark Masses and SM Parameters.

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## 1. Introduction

Apart from dark matter and dark energy, the experimental observation of neutrino oscillations is the only evidences of physics beyond the Standard Model (SM). A global fit of all neutrino data at  $2\sigma$  gives the following allowed ranges for the lepton mixing angles [1]:

$$0.26 \leq \sin^2 \theta_{12} \leq 0.36, \quad 0.38 \leq \sin^2 \theta_{23} \leq 0.63, \quad \sin^2 \theta_{13} \leq 0.025. \quad (1.1)$$

These values are consistent with an especial simple, the so called tribimaximal (TB) mixing ansatz [2]:

$$\sin^2 \theta_{12} = 1/3, \quad \sin^2 \theta_{23} = 1/2, \quad \sin^2 \theta_{13} = 0. \quad (1.2)$$

As has been shown in [3–17] this peculiar mixing pattern can be explained by an  $A_4$  flavor symmetry where  $A_4$  is the discrete group of even permutations among four objects. The  $A_4$  group has four irreducible representations, namely a triplet  $\mathbf{3}$  and three singlets  $\mathbf{1}$ ,  $\mathbf{1}'$  and  $\mathbf{1}''$  [18, 19]. In the majority of the models based on  $A_4$  as flavor symmetry, the three lepton doublets are assigned to the triplet representation of  $A_4$  and the three right-handed singlets are assigned to the three  $A_4$  singlets.

In spite of the success of the  $A_4$  symmetry to explain TB mixing, the extension of these models to the quark sector is not straightforward. The simplest way to extend the  $A_4$  flavor symmetry to the quark sector is to adopt the same structure as within the

lepton sector. With such an assignment, up and down-type quark mass matrices are both diagonalized by the same unitary matrix giving rise to the diagonal CKM matrix. As a first approximation this leading order result is acceptable. However, in order to reproduce the correct CKM matrix it is necessary to introduce some subleading terms responsible for inducing correction of order of the Cabibbo angle  $\lambda$  in the quark mass matrices and so in the CKM matrix. Unluckily it has been shown [10] that the requirement that the subleading terms do not spoil the leptonic TB mixing matrix forces the corrections in the CKM mixing matrix to be of  $\mathcal{O}(\lambda^2)$  instead of  $\mathcal{O}(\lambda)$ . Therefore the corrections coming from subleading terms are too small to explain the observed quark mixing angles. Other possibilities to obtain a realistic CKM matrix without spoiling the leptonic TB mixing prediction have been studied in literature. An example of SU(5) grand unified case can be found in ref. [20]. Models in which the  $A_4$  flavor symmetry both explicitly [21] and spontaneously broken [22], can also be found in literature. Another possibility is to consider the discrete groups larger than  $A_4$  for flavor symmetry. Recently, some models have been proposed based on the discrete symmetries  $T'$  [23, 24] and  $\Delta(27)$  [25, 26].

In this paper, we propose a model based on  $A_4$  adopting a different assignment of the representation for the quarks and leptons. In particular we assign both left and right-handed quarks to the singlets 1,  $1'$  and  $1''$  of  $A_4$ . Therefore, we have enough freedom to make the quark sector quite realistic.

As in [24], in our model only the top quark interaction is present at renormalizable level and we also assume the Higgs doublets superfields  $\hat{H}_u$  and  $\hat{H}_d$  transform as singlets under  $A_4$ . We use lepton number conservation to make a difference between quark and lepton sectors. The charged lepton and quark mass hierarchies are obtained in our model by introducing two auxiliary discrete symmetries  $Z_3$  and  $Z_4$ . All the fermion mass hierarchies are realized only by the discrete symmetries. Unfortunately the model we propose can not be directly incorporated into a grand unified gauge theory. For recent attempts in this direction see [20, 26–30]. The choice of assign both left-handed and right-handed quarks to singlets of  $A_4$  is completely different from models previously proposed in [3–5, 10, 21, 22] where left-handed quarks belong to triplet representation of  $A_4$ . Moreover, most of the  $A_4$  and models based on different discrete groups need to invoke additional assumptions to explain fermion mass hierarchies. For example a continuous symmetry a la Froggatt-Nielsen.

The read of this paper is organized as follows. In section 2 we describe the basic features of the model and its field content. In section 3 we study the scalar potential and vevs alignment problem. In appendix we consider all possible sources of corrections to the leading textures and show the stability of the leading order predictions.

## 2. The model

We assign the MSSM matter fields and the additional gauge singlet fields to the group representations of  $A_4 \times Z_3 \times Z_4 \times U_L(1)$  respectively as shown in tables 1 and 2.  $\hat{L}_i$  and  $\hat{Q}_i$  are the lepton and quark electroweak doublet superfields for  $i$ -th generation, the  $\hat{E}_i^c$ ,  $\hat{U}_i^c$  and  $\hat{D}_i^c$  are the lepton and quark electroweak singlet superfields for  $i$ -th generation, the  $\hat{H}_u$

	$\hat{L}_i$	$\hat{E}_1^c$	$\hat{E}_2^c$	$\hat{E}_3^c$	$\hat{Q}_1$	$\hat{Q}_2$	$\hat{Q}_3$	$\hat{U}_1^c$	$\hat{U}_2^c$	$\hat{U}_3^c$	$\hat{D}_1^c$	$\hat{D}_2^c$	$\hat{D}_3^c$	$\hat{H}_u$	$\hat{H}_d$
$A_4$	3	1	1''	1'	1''	1'	1	1''	1'	1	1	1''	1'	1	1
$Z_3$	$\omega$	$\omega^2$	$\omega$	$\omega^2$	$\omega$	$\omega$	1	$\omega$	$\omega$	1	$\omega^2$	$\omega$	$\omega$	1	1
$Z_4$	1	$i$	$-i$	1	1	1	1	1	1	1	1	1	1	1	1
$U_L(1)$	1	-1	-1	-1	0	0	0	0	0	0	0	0	0	0	0

**Table 1:** The representations of the MSSM matter and Higgs fields under the symmetries associated to  $A_4$ ,  $Z_3$  ( $\omega^3 = 1$ ),  $Z_4$  and lepton number  $L$ . The  $\hat{L}_i$  and  $\hat{Q}_i$  denote the lepton and quark electroweak doublet superfields for  $i$ -th generation, the  $\hat{E}_i^c$ ,  $\hat{U}_i^c$  and  $\hat{D}_i^c$  denote the electroweak singlet superfields for  $i$ -th generation, respectively. The  $\hat{H}_u$  and  $\hat{H}_d$  are Higgs electroweak doublet superfields.

	$\hat{\phi}_T$	$\hat{\phi}_S$	$\hat{\xi}_C, \hat{\xi}'_C$	$\hat{\xi}, \hat{\xi}'$	$\hat{\chi}$	$\hat{\theta}$
$A_4$	3	3	1	1	1'	1''
$Z_3$	1	$\omega$	$\omega$	$\omega$	$\omega^2$	$\omega^2$
$Z_4$	1	1	$i$	1	1	1
$U_L(1)$	0	-2	0	-2	0	0

**Table 2:** The representations of the additional gauge singlet fields under the symmetries associated to  $A_4$ ,  $Z_3$ ,  $Z_4$  and lepton number  $L$ . The  $\hat{\phi}_T$ ,  $\hat{\phi}_S$  are  $A_4$  triplets,  $\hat{\xi}_C, \hat{\xi}'_C, \hat{\xi}, \hat{\xi}'$ ,  $\hat{\chi}$  and  $\hat{\theta}$  are  $A_4$  singlet fields.

and  $\hat{H}_d$  are the up and down type Higgs electroweak doublet superfields, respectively. We also introduce new fields, responsible for flavor symmetry breaking, namely  $\hat{\phi}_S$  and  $\hat{\phi}_T$  as triplets, and  $\hat{\xi}_C, \hat{\xi}'_C, \hat{\xi}, \hat{\xi}'$ ,  $\hat{\chi}$  and  $\hat{\theta}$  as singlets of  $A_4$ , respectively. In the following we will refer to the scalars that transform non trivially under the flavor symmetry  $A_4 \times Z_3 \times Z_4$  as *flavons* and analogously we will address the supermultiplets they belong to as *flavon supermultiplets*.

Once we have assumed that  $\hat{H}_u$  and  $\hat{H}_d$  transform as singlets under  $A_4$ , we are forced to put  $\hat{Q}_3$  and  $\hat{U}_3^c$  in the singlet representation  $\mathbf{1}$  of  $A_4$  in order to have a renormalizable interaction for the top quark. The  $\hat{Q}_3 - \hat{D}_3^c$  interaction, as well as all the others Yukawa interactions for the up and down type quarks, arise from higher dimensional operators. This is because we have chosen the charge assignment for the down sector different from the up one:  $\hat{Q}_3$  and  $\hat{D}_3^c$  transform as  $\mathbf{1}$  and  $\mathbf{1}'$  respectively under  $A_4$ . In particular, we will show in the following that quark mixing angles and mass hierarchies are a consequence of the product of the auxiliary symmetry  $Z_3$  and the  $Z_3$  contained in  $A_4$ . The auxiliary  $Z_4$  symmetry is introduced to explain charged lepton mass hierarchies. Lepton number conservation is crucial to distinguish quark and lepton sectors, namely  $\hat{\phi}_S$  and  $\hat{\xi}$  fields interact with leptons and not with quarks. The field  $\xi'_C$  is identical to  $\xi_C$  and the reason for its introduction will be come clear in section 3, where we will discuss the problem of how to guarantee the correct vacuum alignment. Since we can always rotate  $\xi_C, \xi'_C$  and go to the basis in which just one of the two fields interacts with the fermions, we will neglect the terms involving  $\xi'_C$  in all the Yukawa interactions of the next sections.

## 2.1 Leptons

In the lepton sector, there are no Yukawa interactions in the renormalizable superpotential. All interactions arise when subleading contributions are considered:

$$\begin{aligned}
 W_l = & y_e \hat{E}_1^c (\hat{\phi}_T \hat{L}) \hat{H}_d \frac{\hat{\xi}_C \hat{\xi}_C \hat{\xi}_C}{\Lambda^4} + y_\mu \hat{E}_2^c (\hat{\phi}_T \hat{L})' \hat{H}_d \frac{\hat{\xi}_C}{\Lambda^2} + y_\tau \hat{E}_3^c (\hat{\phi}_T \hat{L})'' \hat{H}_d \frac{1}{\Lambda} \\
 & + y_{\nu\xi} \frac{(\hat{L}\hat{L}) \hat{H}_u \hat{H}_u \hat{\xi}}{\Lambda^2} + y_{\nu S} \frac{(\hat{\phi}_S \hat{L}\hat{L}) \hat{H}_u \hat{H}_u}{\Lambda^2} + h.c. .
 \end{aligned} \tag{2.1}$$

Here  $\Lambda$  is the cut-off scale,  $y_i$  are  $\mathcal{O}(1)$  coupling constants and  $(LL)'$ , for instance, stands for the product of two triplets of  $A_4$  that transforms like a  $\mathbf{1}'$  singlet (see appendix A). In eq. (2.1) we have neglected subleading terms of the form  $\hat{E}_3^c (\hat{\phi}_T \hat{L})'' \hat{H}_d \hat{\chi} \hat{\chi} / \Lambda^4$ ,  $\hat{E}_3^c (\hat{\phi}_T \hat{L})'' \hat{H}_d \hat{\theta} \hat{\theta} / \Lambda^4$ . The charged lepton mass matrix is given by

$$M^l = U \cdot \begin{pmatrix} y_e u_{\xi_C}^3 / \Lambda^3 & 0 & 0 \\ 0 & y_\mu u_{\xi_C} / \Lambda & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \frac{v_T v_d}{\Lambda}; \quad U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \tag{2.2}$$

where  $u_{\xi_C}$ ,  $v_T$  and  $v_d$  are the vevs in the scalar component of  $\hat{\xi}_C$ ,  $\hat{\phi}_T$  and  $\hat{H}_d$ , respectively. For the charged lepton mass spectrum, we obtain

$$\frac{m_\mu}{m_\tau} \simeq \frac{u_{\xi_C}}{\Lambda}; \quad \frac{m_e}{m_\tau} \simeq \left( \frac{u_{\xi_C}}{\Lambda} \right)^3, \tag{2.3}$$

and a realistic hierarchy among the charged lepton masses is recovered by assuming

$$\frac{u_{\xi_C}}{\Lambda} \sim \lambda^2, \tag{2.4}$$

with  $\lambda$  the Cabibbo angle. For the absolute scale of the mass matrix of eq. (2.2), assuming  $y_\tau \sim 1$  and  $v_d \sim 100$  GeV, one finds

$$\lambda^3 \lesssim \frac{v_T}{\Lambda} \lesssim \lambda^2. \tag{2.5}$$

From eq. (2.1) we obtain the effective neutrino mass matrix as

$$M_{LL}^\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix} \frac{v_u^2}{\Lambda}; \quad a = y_{\nu\xi} \frac{u_\xi}{\Lambda}, \quad b = y_{\nu S} \frac{v_S}{\Lambda}, \tag{2.6}$$

where  $u_\xi$ ,  $v_S$  and  $v_u$  are the vevs of the scalar component of  $\hat{\xi}$ ,  $\hat{\phi}_S$  and  $\hat{H}_u$ , respectively. In the basis where the charged lepton mass matrix  $M_l$  in eq. (2.2) is diagonal, namely  $\hat{L} \rightarrow U \hat{L}$ , the neutrino mass matrix is diagonalized by the tribimaximal unitary matrix. The new ingredient here is that the charged lepton mass hierarchies is derived from a  $Z_3 \times Z_4$  discrete symmetry and not a Froggatt-Nielsen continuous one.

The scale  $\Lambda$  of the model can be fixed by requiring that the neutrino Yukawa couplings are of order  $\mathcal{O}(1)$ . With the plausible assumption<sup>1</sup> of having  $v_S \sim u_\xi \sim v_T$  eq. (2.6) then leads

$$\frac{v_u^2 v_S}{\Lambda^2} \simeq 1 \text{ eV}, \tag{2.7}$$

i.e.  $\Lambda \simeq 10^{10} - 10^{12}$  GeV.

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<sup>1</sup>This assumption will be justified in the discussion of the potential of the model, see next section.

## 2.2 Quarks

From the assignments in tables 1 and 2, it follows that the only renormalizable interaction in the superpotential is given by

$$W_q^{(0)} = y_t \hat{Q}_3 \hat{U}_3^c \hat{H}_u, \quad (2.8)$$

giving mass to the top quark. Therefore at leading order only the top quark is massive and all the other quarks are massless. We will show in this section that the correct masses and mixing pattern is achieved by considering subleading corrections to the leading superpotential  $W_q^{(0)}$  for quarks in eq. (2.8). In the next section we will show that such a result is stable, i.e. the mass textures do not change once we consider subleading corrections to the superpotential:

$$\begin{aligned} W_q = & W_q^{(0)} + (c_u \hat{Q}_2 \hat{U}_3^c \hat{\theta} \hat{H}_u + c'_u \hat{Q}_3 \hat{U}_2^c \hat{\theta} \hat{H}_u + b_u \hat{Q}_1 \hat{U}_3^c \hat{\chi} \hat{H}_u + b'_u \hat{Q}_3 \hat{U}_1^c \hat{\chi} \hat{H}_u) / \Lambda \\ & + (y_c \hat{Q}_2 \hat{U}_2^c \hat{\theta} \hat{H}_u + a_u \hat{Q}_1 \hat{U}_2^c \hat{\theta} \hat{\chi} \hat{H}_u + a'_u \hat{Q}_2 \hat{U}_1^c \hat{\theta} \hat{\chi} \hat{H}_u) / \Lambda^2 \\ & + y_u \hat{Q}_1 \hat{U}_1^c \hat{\chi} \hat{\chi} \hat{H}_u / \Lambda^2 \\ & + (y_b \hat{Q}_3 \hat{D}_3^c \hat{\theta} \hat{H}_d + c'_d \hat{Q}_3 \hat{D}_2^c \hat{\chi} \hat{H}_d) / \Lambda \\ & + (y_s \hat{Q}_2 \hat{D}_2^c \hat{\theta} \hat{\chi} \hat{H}_d + b'_d \hat{Q}_3 \hat{D}_1^c \hat{\theta} \hat{\chi} \hat{H}_d + b_d \hat{Q}_1 \hat{D}_3^c \hat{\theta} \hat{\chi} \hat{H}_d + c_d \hat{Q}_2 \hat{D}_3^c \hat{\theta} \hat{\theta} \hat{H}_d + a_d \hat{Q}_1 \hat{D}_2^c \hat{\chi} \hat{\chi} \hat{H}_d) / \Lambda^2 \\ & + (a'_d \hat{Q}_2 \hat{D}_1^c \hat{\chi} \hat{\theta} \hat{\theta} \hat{H}_d + y_d \hat{Q}_1 \hat{D}_1^c \hat{\chi} \hat{\chi} \hat{\theta} \hat{H}_d) / \Lambda^3. \end{aligned} \quad (2.9)$$

Here  $y_i, a_i^{(')}, b_i^{(')}$  and  $c_i^{(')}$  are coefficients of order  $\mathcal{O}(1)$  and the following quark mass matrices result

$$M^u = v_u \begin{pmatrix} \frac{u_\chi^2}{\Lambda^2} y_u & \frac{u_\theta u_\chi}{\Lambda^2} a_u & \frac{u_\chi}{\Lambda} b_u \\ \frac{u_\theta u_\chi}{\Lambda^2} a'_u & \frac{u_\theta^2}{\Lambda^2} y_c & \frac{u_\theta}{\Lambda} c_u \\ \frac{u_\chi}{\Lambda} b'_u & \frac{u_\theta}{\Lambda} c'_u & y_t \end{pmatrix}, \quad M^d = v_d \begin{pmatrix} \frac{u_\theta u_\chi^2}{\Lambda^3} y_d & \frac{u_\chi^2}{\Lambda^2} a_d & \frac{u_\chi u_\theta}{\Lambda^2} b_d \\ \frac{u_\theta^2 u_\chi}{\Lambda^3} a'_d & \frac{u_\theta u_\chi}{\Lambda^2} y_s & \frac{u_\theta^2}{\Lambda^2} c_d \\ \frac{u_\theta u_\chi}{\Lambda^2} b'_d & \frac{u_\chi}{\Lambda} c'_d & \frac{u_\theta}{\Lambda} y_b \end{pmatrix}. \quad (2.10)$$

Here  $u_\theta$  and  $u_\chi$  are the vevs of the scalar component of  $\hat{\theta}$  and  $\hat{\chi}$ . In eqs. (2.10) the up and down quark mass matrices are correctly reproduced for a natural choice of the parameters:

$$\frac{u_\chi}{\Lambda} \sim \lambda^3, \quad \frac{u_\theta}{\Lambda} \sim \lambda^2. \quad (2.11)$$

With the choice of eq. (2.11), the quark masses and mixing angles are approximately given by

$$\begin{aligned} m_u &\approx y_u \frac{u_\chi^2}{\Lambda^2} v_u \sim \lambda^6 v_u, & m_c &\approx y_c \frac{u_\theta^2}{\Lambda^2} v_u \sim \lambda^4 v_u, & m_t &\approx y_t v_u \sim v_u, \\ m_d &\approx y_d \frac{u_\theta u_\chi^2}{\Lambda^3} v_d \sim \lambda^8 v_d, & m_s &\approx y_s \frac{u_\theta u_\chi}{\Lambda^2} v_d \sim \lambda^5 v_d, & m_b &\approx \frac{u_\theta}{\Lambda} y_b v_d \sim \lambda^2 v_d, \end{aligned} \quad (2.12)$$

and the CKM matrix elements can be estimated as

$$V_{us} \approx \frac{u_\chi}{u_\theta} \left( \frac{a_d}{y_s} - \frac{a_u}{y_c} \right) \sim \lambda; \quad V_{ub} \approx \frac{u_\chi}{\Lambda} \left( \frac{b_d}{y_b} - \frac{b_u}{y_t} \right) \sim \lambda^3; \quad V_{cb} \approx \frac{u_\theta}{\Lambda} \left( \frac{c_d}{y_b} - \frac{c_u}{y_t} \right) \sim \lambda^2. \quad (2.13)$$

The quark mass hierarchies of eq. (2.12) are in slight contrast with the experimental data. However the free parameters of  $\mathcal{O}(1)$  in the quark mass matrices in eq. (2.10), allow us to fit all the quark masses by admitting a fine tuning of 10% and a reasonable values of  $\tan \beta$ .

	$\hat{\phi}_R^T$	$\hat{\phi}_R^S$	$\hat{\phi}_R^{\prime S}$	$\hat{\xi}_R^C$	$\hat{\phi}_T$	$\hat{\phi}_S$	$\hat{\xi}_C, \hat{\xi}'_C$	$\hat{\xi}, \hat{\xi}'$	$\hat{\chi}$	$\hat{\theta}$
$A_4$	3	3	3	1	3	3	1	1	1'	1''
$Z_3$	1	1	$\omega$	$\omega^2$	1	$\omega$	$\omega$	$\omega$	$\omega^2$	$\omega^2$
$Z_4$	1	1	1	$-i$	1	1	$i$	1	1	1
$U_L(1)$	0	2	4	0	0	-2	0	-2	0	0
$U_R(1)$	2	2	2	2	0	0	0	0	0	0

**Table 3:** The representations of messenger R-supermultiplets and the flavon supermultiplets.

### 3. The potential

In section 2.1 we have seen that the TB mixing matrix is obtained when the triplets  $\hat{\phi}_T$  and  $\hat{\phi}_S$  break  $A_4$  in the direction  $(v_T, v_T, v_T)$  and  $(v_S, 0, 0)$  respectively. Our potential must break  $A_4$  in this direction.

In section 2 we have implicitly assumed that the part of the superpotential that gives rise to the Yukawa Lagrangian of the SM matter fields is given by eq. (2.1). This part was obtained by integrating out some heavy fields. Inspection of eq. (2.1) show that  $W_l$  is charged under a global  $U(1)_R$  symmetry with  $R$ -charge +2 if we assign to the matter supermultiplets  $R$ -charge +1 and to the flavon supermultiplets involved in the effective superpotential  $W_l$   $R$ -charge 0. The global  $U(1)_R$  is the continuous symmetry that is broken to the discrete  $R$ -parity once we include the gaugino masses into the model. We may assume that  $W_l$  and the part of the superpotential that involves all the supermultiplets that transform non trivially under the flavor symmetry  $A_4 \times Z_3 \times Z_4$  has  $R$ -charge +2 and therefore give rise to a Yukawa superpotential as in eq. (2.1) invariant with respect to the continuous  $U(1)_R$ . Since the flavon supermultiplets bring null  $R$ -charge, to build a superpotential with total  $R$ -charge +2 avoiding the spontaneous breaking of the  $R$ -symmetry, we need some messenger fields that carry  $R$ -charge +2 is necessary that the superpotential be linear in these fields. To the flavon supermultiplets of table 2 we therefore add 4 messenger  $R$ -supermultiplets given in table 3. The superpotential  $W$  is invariant under all the symmetries of the model that contain an explicit breaking term of the continuous  $U_L(1)$  and the discrete  $Z_3$  symmetries. We assume that the explicit breaking term arises from the presence of an heavy sector that does not interact with the matter content of the model, thus ensuring that all the terms generated in the Yukawa superpotential by the  $U_L(1)$  and  $Z_3$  explicit breaking terms are sufficiently suppressed and do not affect the fermion mass matrix structures. The spontaneous breaking of the continuous lepton number  $U(1)_L$  gives to unphysical massless Goldstone boson. We will discuss in detail in the appendix the problem of the Goldstone boson. Nevertheless, as shown in the appendix, by imposing the  $R$ -symmetry invariance under the discrete symmetries, our superpotential presents some accidental global continuous symmetries that give massless states. We guarantee the stability of the minimum and give positive mass to the massless states by adequately choosing the  $V_{\text{soft}}$  of SUSY. The full superpotential  $W$  is given by

$$\begin{aligned}
 W = & M_T \hat{\phi}_R^T \hat{\phi}_T + \lambda_T \hat{\phi}_R^T \hat{\phi}_T \hat{\phi}_T + \lambda_{S\theta} (\hat{\phi}_R^S \hat{\phi}_S)' \hat{\theta} + \lambda_{S\chi} (\hat{\phi}_R^S \hat{\phi}_S)'' \hat{\chi} + M_C \hat{\xi}_R^C \hat{\xi}_C + M'_C \hat{\xi}_R^C \hat{\xi}'_C \\
 & + \lambda_S \hat{\phi}_R^{\prime S} \hat{\phi}_S \hat{\phi}_S + \lambda_\xi \hat{\phi}_R^{\prime S} \hat{\phi}_S \hat{\xi} + \lambda'_\xi \hat{\phi}_R^{\prime S} \hat{\phi}_S \hat{\xi}' .
 \end{aligned}
 \tag{3.1}$$

From eq. (3.1) we derive the scalar potential by

$$V = \left| \frac{\partial W}{\partial f_{\phi_i}} \right|^2 + V_{\text{soft}}, \tag{3.2}$$

where  $V_{\text{soft}}$  includes all possible SUSY soft terms for the new scalars of the model invariant under all the discrete ( $A_4, Z_3, Z_4$ ) and the continuous ( $U_L(1)$ ) symmetries. It breaks the accidental global continuous symmetries presented by the SUSY invariant scalar potential. We will discuss this point in detail in the appendix. Since the scale of  $V_{\text{soft}}$  is  $1 - 10$  TeV, while the scale of the SUSY invariant potential is  $10^{10} - 10^{12}$  GeV, we can neglect  $V_{\text{soft}}$  and search for a vacuum configuration that is SUSY invariant. It is given by

$$\begin{aligned} \langle \phi_T \rangle &= (v_T, v_T, v_T), \\ \langle \phi_S \rangle &= (v_S, 0, 0), \\ \langle \chi \rangle &= u_\chi, \\ \langle \xi \rangle &= u_\xi, \\ \langle \xi' \rangle &= u'_\xi, \\ \langle \xi_C \rangle &= u_{\xi_C}, \\ \langle \xi'_C \rangle &= u_{\xi'_C}, \\ \langle \theta \rangle &= u_\theta. \end{aligned} \tag{3.3}$$

Next we want to identify the region of the parameter space for which the vacuum configuration of eq. (3.3) is the minimum of eq. (3.2) with  $V_{\text{soft}} = 0$ . Since none of the messenger fields acquires a vev and since they enter linearly in all the terms of the superpotential, in the SUSY limit all the derivatives with respect to the  $F$  components of the supermultiplets not charged with respect to the  $U(1)_R$  symmetry vanish. Therefore in the discussion of the vacuum configuration we have to take into account only the derivatives with respect to the  $F$  components of the messenger supermultiplets. Taking the derivative of  $W$  with respect to the  $F$  components of the supermultiplets  $\hat{\phi}_R^T, \hat{\phi}_R^S$  and  $\hat{\phi}_R^{\prime S}$  and substituting the vacuum configuration of eq. (3.3) we find

$$\begin{aligned} \frac{\partial W}{\partial f_{\phi_{Ri}^T}} &= \frac{M_T}{3} v_T + \frac{1}{3} \lambda_T v_T^2, \\ \frac{\partial W}{\partial f_{\phi_{R1}^S}} &= \frac{\lambda_{S\theta}}{3} v_S u_\theta + \frac{\lambda_{S\chi}}{3} v_S u_\chi, \\ \frac{\partial W}{\partial f_{\phi_{R2,3}^S}} &= 0, \\ \frac{\partial W}{\partial f_{\phi_{R1}^{\prime S}}} &= \lambda_\xi v_S u_\xi + \lambda'_\xi v_S u'_\xi \\ \frac{\partial W}{\partial f_{\phi_{R1,2}^{\prime S}}} &= 0. \end{aligned} \tag{3.4}$$



From eq. (3.4) one finds that a possible solution conserving SUSY is given by

$$\begin{aligned}
 v_T &= -\frac{M_T}{\lambda_T}, \\
 u_\chi &= -\frac{\lambda_{S\theta}}{\lambda_{S\chi}} u_\theta, \\
 u_\xi &= -\frac{\lambda'_\xi}{\lambda_\xi} u'_\xi.
 \end{aligned}
 \tag{3.5}$$

Taking the derivative with respect to the  $F$  component of  $\hat{\xi}_R^C$  we have

$$\frac{\partial W}{\partial f_{\xi_R^C}} = \lambda_C u_{\xi_C} + \lambda'_C u_{\xi'_C},
 \tag{3.6}$$

from which it follows that

$$u_{\xi_C} = -\frac{\lambda'_C}{\lambda_C} u_{\xi'_C}.
 \tag{3.7}$$

A few comments might be in order. In the sections 2.1–2.2 we have seen that the correct fermion mass matrices are obtained assuming that the vevs satisfy

$$\begin{aligned}
 \frac{u_\theta}{\Lambda} &\sim \frac{u_{\xi_C}}{\Lambda} \sim \lambda^2, \\
 \frac{v_T}{\Lambda} &\sim \frac{v_S}{\Lambda} \sim \frac{u_\xi}{\Lambda} \sim \lambda^2 \div \lambda^3, \\
 \frac{u_\chi}{\Lambda} &\sim \lambda^3,
 \end{aligned}
 \tag{3.8}$$

with  $\lambda$  the Cabibbo angle.

From eqs. (3.5) we see that in order to satisfy the relations given in eq. (3.8) we need  $M_T \sim \lambda^2 \Lambda$ ,  $\frac{\lambda_{S\theta}}{\lambda_{S\chi}} \sim \lambda$ . Aside from  $v_T$ , all the vevs remain undetermined and their natural value tends to be  $\Lambda$  (the cut-off scale) and not  $(\lambda^2 \div \lambda^3)\Lambda$ . The problem of how to stabilize the relations given in eq. (3.8) is therefore not completely solved. We assume that it can be solved by including loop-contributions or explicit breaking terms of the abelian discrete symmetry arising in a hidden scalar sector. We do not enter into the details of this problem and assume that there exists a choice of the parameters of the potential that satisfy the relations given in eq. (3.8). Once we include  $V_{\text{soft}}$  we have to choose the soft terms in such a way that  $V_{\text{soft}} < 0$  in the vacuum configuration of eq. (3.8). The stability of the minimum of the potential of eq. (3.2) can be assured.

Corrections to the leading mass matrix textures of section 2 are induced by higher order operators in the superpotential of eq. (3.1) that can change the vacuum configuration given in eq. (3.3). These possibility could disalign the triplet vevs or may be directly affect the mass matrices. In appendix we accomplish the full analysis of higher order corrections to the mass matrix textures. In particular we check that the corrections induced by the scalar interactions and by the introduction of higher order operators in the superpotential are under control and do not destroy the leading order predictions of the model.

## 4. Conclusions

We have proposed a supersymmetric extension of the standard model based on the discrete flavor group  $A_4$ . The new features with respect to earlier work present in the literature is that all the quarks, both left-handed and right-handed, transform as singlet representations of  $A_4$ ,  $\mathbf{1}$ ,  $\mathbf{1}'$  and  $\mathbf{1}''$ . This assignment allows us to ensure that only the top quark acquires mass at tree level. All other entries in the mass matrices are generated by higher dimensional operators and are suppressed by powers of  $1/\Lambda$ . The introduction of two auxiliary discrete symmetries  $Z_3$  and  $Z_4$  have allowed us to obtain realistic charged fermion mass hierarchies and the CKM mixing matrix, without appealing to a continuous  $U(1)_F$  flavor symmetry, lepton number avoids dangerous mixing between quarks and leptons. Finally, we have studied the scalar superpotential that presents the correct  $A_4$  breaking alignments and with the introduction of apposite explicit breaking terms of the abelian symmetry of the model we have determined all the flavon scalar vevs.

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## A. Conventions

The finite group of the even permutations of four objects is  $A_4$  [18, 19]. Its generators  $S$  and  $T$  obey the relations

$$S^2 = (ST)^3 = T^3 = 1.$$

We remark that  $A_4$  has four irreducible representations,  $\mathbf{3}$ ,  $\mathbf{1}$ ,  $\mathbf{1}'$  and  $\mathbf{1}''$  satisfying the following product rule

$$\begin{aligned} \mathbf{3} \times \mathbf{3} &= \mathbf{3} + \mathbf{3} + \mathbf{1} + \mathbf{1}' + \mathbf{1}'' \\ \mathbf{1} \times \mathbf{1} &= \mathbf{1}, \quad \mathbf{1}' \times \mathbf{1}'' = \mathbf{1}, \quad \mathbf{1}' \times \mathbf{1}' = \mathbf{1}'', \dots \end{aligned} \tag{A.1}$$

We chose to work in the basis in which  $S$  is a diagonal matrix in the three-dimensional representation. Different basis, like the one where  $T$  is diagonal, are related by unitary transformations, see for instance [15]. Therefore if  $a = (a_1, a_2, a_3)$  and  $b = (b_1, b_2, b_3)$  are two triplets of  $A_4$  their products are given by

$$\begin{aligned} \mathbf{1} &= (ab) = \frac{1}{\sqrt{3}}(a_1b_1 + a_2b_2 + a_3b_3), \\ \mathbf{1}' &= (ab)' = \frac{1}{\sqrt{3}}(a_1b_1 + \omega^2a_2b_2 + \omega a_3b_3), \\ \mathbf{1}'' &= (ab)'' = \frac{1}{\sqrt{3}}(a_1b_1 + \omega a_2b_2 + \omega^2a_3b_3), \\ \mathbf{3} &= (ab)_3 = \frac{1}{\sqrt{3}}(a_2b_3, a_3b_1, a_1b_2), \\ \mathbf{3}' &= (ab)'_3 = \frac{1}{\sqrt{3}}(a_3b_2, a_1b_3, a_2b_1). \end{aligned} \tag{A.2}$$

## B. The scalar potential

The full expression of the scalar potential at the leading order is given by eq. (3.2)

$$V = V_{\text{SUSY}} + V_{\text{soft}}. \quad (\text{B.1})$$

Here  $V_{\text{soft}}$  contains soft SUSY breaking terms that stabilize the minimum of  $V_{\text{SUSY}}$ . From eqs. (3.1) the part of the scalar potential that involves only the flavons is given by

$$V_{\text{SUSY}} = |M_T \phi_T + 2\lambda \phi_T \phi_T|^2 + |\lambda_{S\theta} \phi_S \theta + \lambda_{S\chi} \phi_S \chi|^2 \\ + |\lambda_S \phi_S \phi_S + \lambda'_\xi \phi_S \xi' + \lambda_\xi \phi_S \xi|^2 + |M_C \xi_C + M'_C \xi'_C|^2. \quad (\text{B.2})$$

Without loss of generality, we can rotate the  $\xi$  and  $\xi'$  fields in new combinations  $\tilde{\xi}'$  and  $\tilde{\xi}$  in which only  $\tilde{\xi}'$  develop a vev. The same argumentation can be applied to  $\xi_C$  and  $\xi'_C$  obtaining  $\tilde{\xi}_C$  and  $\tilde{\xi}'_C$ . In terms of the new fields the potential of eq. (B.2) is given by

$$V_{\text{SUSY}} = |M_T \phi_T + 2\lambda \phi_T \phi_T|^2 + |\lambda_{S\theta} \phi_S \theta + \lambda_{S\chi} \phi_S \chi|^2 \\ + |\lambda_S \phi_S \phi_S + \tilde{\lambda}_\xi \phi_S \tilde{\xi}|^2 + |\tilde{M}_C \tilde{\xi}_C|^2. \quad (\text{B.3})$$

with  $\tilde{\lambda}_\xi = \sqrt{\lambda_\xi^2 + \lambda'^2_\xi}$  and  $\tilde{M}_C = \sqrt{M_C^2 + M'^2_C}$ . At this level the two combination that develop vevs,  $\tilde{\xi}'$  and  $\tilde{\xi}_C$ , are massless, lacking of a potential. They acquire a (positive) mass only when we add to the SUSY scalar potential of eq. (B.3) the soft scalar potential  $V_{\text{soft}}$ , provided it contains adequate mass terms, as it has been done in eq. (B.13) below. The rotation done for  $\xi, \xi'$  and  $\xi_C, \xi'_C$  is not allowed for  $\chi$  and  $\theta$  since this two scalar fields behave differently under  $A_4$ . The terms given by

$$|\lambda_S \phi_S \phi_S + \tilde{\lambda}_\xi \phi_S \tilde{\xi}|^2 + |\lambda_{S\theta} \phi_S \theta + \lambda_{S\chi} \phi_S \chi|^2, \quad (\text{B.4})$$

present two accidental continuous global symmetry. The first term in eq. (B.4)

$$|\lambda_S \phi_S \phi_S + \tilde{\lambda}_\xi \phi_S \tilde{\xi}|^2$$

presents an accidental continuous symmetry  $\text{SO}(3) \times \text{U}(1)_L \sim O(3)$ . With respect to  $\text{SO}(3)$ ,  $\phi_S$  transforms as a triplet, while  $\tilde{\xi}, \chi$  and  $\theta$  as singlets. When  $\phi_S$  acquires vev as  $\langle \phi_S \rangle = (v_S, 0, 0)$  the accidental continuous symmetry is broken to  $\text{SO}(2) \times \text{U}(1) \sim O(2)$  leaving two massless Goldstone bosons. Above the two fields charged under the lepton number  $\text{U}(1)_L$ ,  $\phi_S$  and  $\tilde{\xi}$ , only  $\phi_S$  develops a vev. As consequence, the Goldstone boson associated to the breaking of the abelian  $\text{U}(1)_L$  has projection only along  $\phi_S$  as can be seen in eq. (B.5). The real parts of  $\chi$  and  $\theta$  belong to a doublet of  $O(2)$  and the imaginary components of  $\chi$  and  $\theta$  belong to another doublet of  $O(2)$ . When  $\chi$  and  $\theta$  develop vevs, the  $O(2)$  global symmetry is broken and other two massless Goldstone bosons arise. Indeed the mass matrices for the real and imaginary components of the scalars involved in the part of the scalar potential of eq. (B.4) are identical. In the basis  $(\phi_{S_1}, \phi_{S_2}, \phi_{S_3}, \tilde{\xi}, \chi, \theta)$

the mass matrix is given by

$$\mathcal{M}_S^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{2}{3}(\lambda_S v_S^2 + \lambda_{S\theta} u_\theta^2) & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{3}(\lambda_S v_S^2 + \lambda_{S\theta} u_\theta^2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{3}(\lambda_\xi^2 + \lambda_\xi'^2)v_S^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{9}\lambda_{S\chi}^2 v_S^2 & \frac{2}{9}\lambda_{S\chi}\lambda_{S\theta} v_S^2 \\ 0 & 0 & 0 & 0 & \frac{2}{9}\lambda_{S\chi}\lambda_{S\theta} v_S^2 & \frac{2}{9}\lambda_{S\theta}^2 v_S^2 \end{pmatrix}. \quad (\text{B.5})$$

We recognize 1 + 1 Goldstone bosons from the entry (1,1) of the matrix  $\mathcal{M}_S^2$ . The determinant of the sub-block 5,6 of the matrix  $\mathcal{M}_S^2$  is zero, so  $\mathcal{M}_S^2$  gives other 1 + 1 Goldstone bosons.

The first term in the potential of eq. (B.2) is

$$|M_T \phi_T + 2\lambda \phi_T \phi_T|^2. \quad (\text{B.6})$$

It is invariant under an accidental SO(3) symmetry under which  $\phi_T$  transform as a triplet. When  $\phi_T$  acquire vev along the direction (1, 1, 1), SO(3) is broken into  $S_3$  giving rise to the following mass matrix for the real (imaginary) components of  $\phi_T$

$$\mathcal{M}_T^2 = 8\lambda_T^2 v_T^2 \begin{pmatrix} 1 & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & 1 & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \end{pmatrix}. \quad (\text{B.7})$$

Since the mass matrix of eq. (B.7) is  $S_3$  invariant the mass eigenstates are one singlet of  $S_3$ , with mass given by  $\frac{8}{3}\lambda_T^2 v_T^2$ , and one  $S_3$  doublet, with mass  $\frac{32}{3}\lambda_T^2 v_T^2$ . The absence of Goldstone bosons suggests that when  $\phi_T$  acquires vev in the direction (1, 1, 1) the continuous accidental global symmetry SO(3) is broken to a continuous group of its same dimensions, 3, and that contains  $S_3$ . This group is SU(2) and the doublet of  $S_3$  transforms as a doublet of SU(2).

The scalar potential of eq. (B.2) gives in total 4 + 4 massless particles. We break explicitly the accidental global symmetries and the continuous lepton number U(1)<sub>L</sub> in  $V_{\text{soft}}$  in order to give masses to these 8 massless particles. The breaking terms have to provide positive mass to the unwanted Goldstone bosons and in general this is not an easy issue since  $V_{\text{soft}}$  has to be negative in the minimum in order to guarantee the stability of the potential. Indeed the problem related to the presence of flavor Goldstone bosons is given by the 4-fermion effective operator [31, 32]. This effective operator may be parametrized by

$$\frac{1}{m_\alpha^2} \frac{m_{ij}^f m_{kl}^f}{v_F^2} \bar{\psi}_i (c_V^{ij} + \gamma_5 c_A^{ij}) \psi_j \bar{\psi}_k (c_V^{kl} + \gamma_5 c_A^{kl}) \psi_l \quad (\text{B.8})$$

where  $m_{ij}^f$  is the fermion mass matrix entry,  $v_F$  the flavor symmetry breaking scale and  $m_\alpha$  the mass of the scalar or pseudoscalar flavon that mediates the process that gives rise to the 4-fermion operator. If we want to give a very rough and conservative estimate of the

mass  $m_\alpha$  needed to suppress flavor changing processes we can overestimate the operator of eq. (B.8) with

$$\frac{1}{m_\alpha^2} \frac{m_3^{f2}}{v_F^2} \bar{\psi}_i \psi_j \bar{\psi}_k \psi_l, \quad (\text{B.9})$$

where  $m_3^f$  is the heaviest mass in the  $f$  fermion family. If we now consider for example the process  $\mu \rightarrow 3e$  we obtain that the operator given in eq. (B.9) gives a decay width

$$\Gamma_{\mu \rightarrow 3e} \sim \frac{1}{16\pi^3} \frac{m_\mu^5 m_\tau^4}{m_\alpha^4 v_F^4}, \quad (\text{B.10})$$

and by comparing eq. (B.10) with

$$\Gamma_{\mu \rightarrow all} = \frac{m_\mu^5 G_F^2}{192\pi^3}, \quad (\text{B.11})$$

and with the experimental bound  $\Gamma_{\mu \rightarrow 3e}/\Gamma_{\mu \rightarrow all} < 10^{-12}$ , we obtain for a flavor breaking scale around  $10^{10}$  GeV the very low bound

$$m_\alpha \sim 1\text{GeV}. \quad (\text{B.12})$$

The same bound is obtained by considering other tree level flavor changing processes like  $K - \bar{K}$  oscillation. If we assume that the SUSY breaking scale of the soft potential  $V_{\text{soft}}$  is around the TeV, then the inclusion in  $V_{\text{soft}}$  of terms that give a positive mass around the GeV to the unwanted Goldstone bosons does not endanger the stability of the potential. However we lack of a dynamical principle to justify why the scale of some soft terms is around the TeV while that of the others is around the GeV and therefore we prefer assuming that all the soft terms are around 100 GeV- 1 TeV and imposing the constrain  $V_{\text{soft}} < 0$  on the parameter of  $V_{\text{soft}}$ . For example the soft potential  $V_{\text{soft}}$

$$V_{\text{soft}} = \frac{m_T^2}{2} (\phi_T^2 + H.c.) + \frac{A_T}{3} (\phi_T^3 + H.c.) + \tilde{m}_T^2 |\phi_T|^2 + A_S [(\phi_S \phi_S)' \chi + H.c.] + m_{\tilde{\xi}'_C}^2 |\tilde{\xi}'_C|^2 + m_{\tilde{\xi}'_I}^2 |\tilde{\xi}'_I|^2, \quad (\text{B.13})$$

contains explicitly mass terms for  $\tilde{\xi}'_C$  and  $\tilde{\xi}'_I$ . At the same time the term  $A_S(\phi_S \phi_S)' \chi$  breaks the lepton number, the accidental  $\text{SO}(3)$ , since  $\chi$  is not a singlet of  $\text{SO}(3)$ , and also the accidental  $O(2)$  symmetry involving  $\chi$  and  $\theta$ . If we assume the positivity of all the vevs, we ensure the stability of the potential and the positivity of the pseudo-Goldstone boson masses when the soft terms  $m_T^2, \tilde{m}_T^2, A_T$  are negative while the soft terms  $A_S, m_{\tilde{\xi}'_C}^2, m_{\tilde{\xi}'_I}^2$  positive and holds the condition

$$(m_T^2 + \tilde{m}_T^2 + 2A_T u_T) u_T^2 + A_S u_S^2 u_\chi + m_{\tilde{\xi}'_C}^2 u_{\tilde{\xi}'_C}^2 + m_{\tilde{\xi}'_I}^2 u_{\tilde{\xi}'_I}^2 < 0. \quad (\text{B.14})$$

In this way we guarantee the existence of a large region in the parameter space for which the configuration of eq. (3.3) is the minimum of the potential of eq. (3.2) and avoid the presence of massless particles. Notice that the soft term

$$A_S [(\phi_S \phi_S)' \chi + H.c.]$$

breaks also the discrete  $Z_3$  symmetry a part from the continuous leptonic number  $\text{U}(1)_L$ . We can not care for its effects since the scale of  $V_{\text{soft}}$  is many order of magnitude smaller than the scale of the potential of eq. (3.3).

## C. Corrections to the mass matrix textures

In the following we will discuss the corrections to the leading mass matrix textures of section 2. The corrections to the leading textures of section 2 are induced by higher order operators in the superpotential of eq. (3.1) and are of two kinds:

- C.1** that change the vacuum configuration given in eq. (3.3) and in particular disalign the triplets vevs; that directly affect the mass matrices, see sub-section C.1;
- C.2** that directly affect the mass matrices, see sub-section C.2.

### C.1 Effects in the flavon superpotential

In this section we address the problem of the inclusion of the next to leading order operators in the superpotential of eq. (3.1). We consider only the next to leading order operators that respect all the flavor symmetry  $A_4 \times Z_3 \times Z_4$  and the continuous lepton number  $U(1)_L$  since all the explicit breaking terms appear in  $V_{\text{soft}}$  at a scale which is many order of magnitude lower than the flavor breaking scale.

To the full leading order superpotential  $W$  given in of eq. (3.1), we add the next to leading order part given by

$$W_{NL} = W_{NL_T} + W_{NL_S} + W_{NL_{S'}}, \quad (\text{C.1})$$

where the different terms read as

$$\begin{aligned} W_{NL_T} &= \lambda_{T_1} \frac{1}{\Lambda} (\hat{\phi}_R^T \hat{\phi}_T) (\hat{\phi}_T \hat{\phi}_T) + \lambda_{T_2} \frac{1}{\Lambda} (\hat{\phi}_R^T \hat{\phi}_T)' (\hat{\phi}_T \hat{\phi}_T)'' + \lambda_{T_3} \frac{1}{\Lambda} (\hat{\phi}_R^T \hat{\phi}_T)'' (\hat{\phi}_T \hat{\phi}_T)', \\ &\quad + \lambda_{T_4} \frac{1}{\Lambda} (\hat{\phi}_R^T \hat{\phi}_T \hat{\phi}_T)_3 \hat{\phi}_T, \\ W_{NL_S} &= \lambda_{S_1} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_S \hat{\phi}_T)' \hat{\theta} + \lambda_{S_2} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_S \hat{\phi}_T)'' \hat{\chi} + \lambda_{S_3} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_T)'' \hat{\xi} \hat{\chi} + \lambda_{S_4} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_T)' \hat{\xi} \hat{\theta} \\ &\quad + \lambda'_{S_3} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_T)'' \hat{\xi}' \hat{\chi} + \lambda'_{S_4} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_T)' \hat{\xi}' \hat{\theta}, \\ W_{NL_{S'}} &= \lambda_{\xi_1} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_S \hat{\phi}_S \hat{\phi}_T) + \lambda_{\xi_2} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_S \hat{\phi}_T) \hat{\xi} + \lambda'_{\xi_2} \frac{1}{\Lambda} (\hat{\phi}_R^S \hat{\phi}_S \hat{\phi}_T) \hat{\xi}'. \end{aligned} \quad (\text{C.2})$$

The new superpotential is given by

$$W' = W + W_{NL} \quad (\text{C.3})$$

and the scalar potential with  $V_{\text{soft}} = 0$  is now

$$V = \left| \frac{\partial W'}{\partial f_{\phi_i}} \right|^2. \quad (\text{C.4})$$

The effect of  $W_{NL}$  on the SUSY-conserving vacuum configuration of eq. (3.3) is just a shift in the vevs of the scalar fields and therefore the new vacuum configuration is given by

$$\begin{aligned} \langle \phi_T \rangle &= (v_T + \delta v_{T_1}, v_T + \delta v_{T_2}, v_T + \delta v_{T_3}), \\ \langle \phi_S \rangle &= (v_S + \delta v_{S_1}, \delta v_{S_2}, \delta v_{S_3}), \\ \langle \varphi_i \rangle &= u_i + \delta u_i, \end{aligned} \quad (\text{C.5})$$

where with  $\varphi_i$  we have indicated the generic  $A_4$  singlet and with  $u_i, \delta u_i$  its leading order vev and the shift of the vev respectively. Since we expect the shifts to be subleading and scale as  $\mathcal{O}(1/\Lambda)$ , given the conditions

$$\frac{\partial W'}{\partial f_{\phi_i}|_{v_i+\delta v_i}} = 0, \quad (\text{C.6})$$

obtained by imposing a SUSY conserving vacuum, we expand them linearly in the shifts

$$\frac{\partial W}{\partial f_{\phi_i}|_{v_i+\delta v_i}} + \frac{\partial W_{NL}}{\partial f_{\phi_i}|_{v_i}} = 0, \quad (\text{C.7})$$

and determine the shifts by solving the system given by eq. (C.7). For simplicity we give only the approximate solutions obtained in the limit in which all the couplings  $\lambda_i$  are equal. The derivatives with respect to the components  $f_{\phi_{R_i}^T}$  give the following conditions

$$\begin{aligned} \delta v_{T_1} &= \delta v_{T_2} = \delta v_{T_3} = \delta v_T, \\ \delta v_T &\sim \frac{v_T^2}{\Lambda}. \end{aligned} \quad (\text{C.8})$$

The derivatives with respect to the components  $f_{\phi_{R_i}^S}$  give

$$\begin{aligned} \delta v_{S_2} &= \delta v_{S_3}, \\ \delta v_{S_1} &\sim \frac{u_\chi u_\xi}{\Lambda}, \\ \delta v_{S_2} &\sim \frac{v_T u_\xi}{\Lambda}. \end{aligned} \quad (\text{C.9})$$

Finally the last derivatives give

$$\delta u_\xi \sim \delta u_{\xi'} \sim \frac{v_S v_T + v_T u_\xi}{\Lambda}. \quad (\text{C.10})$$

For the singlet  $\xi, \xi'$  of  $A_4$  we have to ensure that  $\delta u_\xi/\Lambda \leq u_\xi/\Lambda$  ( $\delta u_{\xi'}/\Lambda \leq u_{\xi'}/\Lambda$ ) and it is straightforward to check. For the triplets we need that the effects of the vev disalignments due to the presence of the shifts do not destroy the TB predictions, or in other words, the corrections induced in  $\sin \theta_{23}, \sin \theta_{12}, \sin \theta_{13}$  still keep in the interval indicated by the experimental data at the  $3 - \sigma$  level. As shown in [10] this constrains impose shifts that disalign the vevs to be of order

$$\frac{\delta v}{\Lambda} \leq \lambda^2 v, \quad (\text{C.11})$$

with  $v$  the vev of the generic triplet. In our case the shift of  $\langle \phi_T \rangle$  maintains the correct alignment, so we only have to worry about the disalignment of  $\langle \phi_S \rangle$ . Since at the leading order its vev alignment is given by  $(1, 0, 0)$ , then we have only to consider  $\delta v_{S_2}$  and  $\delta v_{S_3}$ . eqs. (C.8)–(C.9) and the assumptions done in the previous sections lead to

$$\frac{\delta v_{S_{2,3}}}{\Lambda} \sim \lambda^3, \quad (\text{C.12})$$

that satisfy the conditions in eq. (C.11).

## C.2 Effects in the Yukawa superpotential

Giving a look to the mass matrices of eq. (2.2), (2.6), (2.10) we see that highest operators giving contribute to mass matrices have scale as power of  $1/\Lambda^3$ . We have neglected operators of order greater than  $1/\Lambda^3$ . To be consistent we should consider if the inclusion of higher order operators may destroy the fermion mass hierarchies and the quark and lepton mixing matrices. For what concerns the quark sector the answer is trivial. For the up quark the general entry may be written as

$$W_q = \hat{Q}_a \hat{U}_b^c \hat{H}_u \left( \lambda_{ab} \left( \frac{\hat{\chi}}{\Lambda} \right)^n \left( \frac{\hat{\theta}}{\Lambda} \right)^m + \lambda'_{ab} \left( \frac{\hat{\theta}}{\Lambda} \right)^r \left( \frac{\hat{\xi}_C^2}{\Lambda^2} \right)^s \right). \quad (\text{C.13})$$

Here  $n, m, r$  and  $s$  are integers and  $a, b$  are flavor indexes. We can obtain a similar structure for the down quark. Giving a look to the field content of table 2 we see that the conservation of the lepton number in the matter sector imposes that the first higher order operators are obtained by substituting  $\hat{\chi}$  in eq. (C.13) with  $(\hat{\phi}_T \hat{\phi}_T)'' \hat{\theta} / \Lambda^2$  or  $\hat{\theta}$  with  $(\hat{\phi}_T \hat{\phi}_T)' \hat{\chi} / \Lambda^2$ . Since  $u_\chi \sim \lambda u_\theta$  the latter is automatically suppressed. However also the first operator gives a very suppressed correction since it goes as  $v_T^2 u_\theta / \Lambda^2 \sim \lambda^4 u_\chi$ .

For the charged leptons the general Yukawa superpotential may be written as

$$W_l = y_a \hat{E}_L^{c_a} \frac{(\hat{\phi}_T \hat{L}_L)^{r_a}}{\Lambda} \hat{H}_d \left( \frac{\hat{\xi}_C}{\Lambda} \right)^n, \quad (\text{C.14})$$

that represents the compact expression of eq. (2.1). Here  $n$  is an integer,  $a$  is the flavor index and  $r_a$  indicates the different  $A_4$  contractions of the triplets  $\hat{\phi}_T$  and  $\hat{L}_L$  that combine with the different  $A_4$  singlets  $\hat{E}^{c_a}$  to give an invariant. Due to the conservation of the  $Z_4$  symmetry at order  $1/\Lambda^{2+n}$  there is only one operator given by

$$\delta W_l = y_a \hat{E}_L^{c_a} \frac{(\hat{\phi}_T \hat{\phi}_T \hat{L}_L)^{r_a}}{\Lambda^2} \hat{H}_d \left( \frac{\hat{\xi}_C}{\Lambda} \right)^n. \quad (\text{C.15})$$

This operator is potentially dangerous because changes the structure of the charged lepton mass matrix and as consequence the charged lepton mass matrix is no longer diagonalized by the  $U$  given in eq. (2.2) but by a  $U' = U + \delta U$ . This implies that the lepton mixing matrix is deviated by the TB by an amount of order  $\mathcal{O}(\delta U)$ . However, since  $\delta U \sim \mathcal{O}(v_T^2 / \Lambda^2)$ , we obtain the relation  $\delta U / U \sim \mathcal{O}(v_T / \Lambda) \sim \mathcal{O}(\lambda^2 \div \lambda^3)$ . A deviation from the TB of this order is still compatible with the experimental data as indicated in the previous section.

For the neutrinos the general entry is given by

$$W_l = y_{\nu\xi} \frac{(\hat{L}\hat{L}) \hat{H}_u \hat{H}_u \hat{\xi}}{\Lambda^2} + y_{\nu S} \frac{(\hat{\phi}_S \hat{L}\hat{L}) \hat{H}_u \hat{H}_u}{\Lambda^2} + H.c. \quad (\text{C.16})$$

and again the lepton number conservation implies that the only term that can give some dangerous corrections is given by

$$\delta W_l = \frac{\hat{\xi} \hat{\phi}_T (\hat{L}\hat{L}) \hat{H}_u \hat{H}_u}{\Lambda^3}. \quad (\text{C.17})$$

The same arguments used for the charged lepton mass matrix are applied here since the deviation from the TB induced by eq. (C.17) is again of order  $\mathcal{O}(v_T / \Lambda) \sim \mathcal{O}(\lambda^2 \div \lambda^3)$ , being  $u_\xi \sim v_S$ .



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